A NEW METHOD FOR SOLVING DISCONTINUITY PROBLEMS IN MICROSTRIP LINES

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Abstract

A new method has been developed for computing the fringe capacitance of the junctions in microstrip lines. It utilizes Galerkin's method in the spectral domain and is numerically more efficient than many conventional methods.

Introduction

Although the modal fields in the microstrip lines are of hybrid nature,[1] it was proved that at low frequencies the quasi-TEM approximation holds very well and is, in fact, very practical in designing such transmission lines.[2,3] In the actual microwave IC design, one normally encounters several finite sections and various types of junctions of microstrip lines. Hence, it is very important to establish accurate methods for solving these structures and thereby to derive a reliable design procedure.

In this paper, excess or fringing capacitance of finite or semi-infinite sections of microstrip line was calculated at low operating frequencies using an entirely new technique. It is based on the formulation and the derivation of the matrix equation in the spectral domain as opposed to the conventional space domain analysis. The derived algebraic equation was solved using Galerkin's method in the spectral domain.

Theory

Figure 1 shows a typical finite section of microstrip line. In the low frequency range, the Poisson equation is solved for the potential functions $\phi(x,y,z)$ in the structure. To this end, the Poisson equation for the charge and potential is Fourier transformed with respect to both the x and z directions. The transform is defined by

$$\tilde{\phi}(\alpha,y,\beta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi(x,y,z) \, e^{j(\alpha x + \beta z)} dx dz \qquad (1)$$

The potentials of the center strip and the ground plane are assumed to be one and zero volts, respectively. The boundary and continuity conditions in the transformed domain are

(i) $\tilde{\phi}(\alpha,-b,\beta) = 0$

(ii) $\tilde{\phi}(\alpha,y,\beta) \to 0$ as $y \to +\infty$

(iii) $\tilde{\phi}(\alpha,0+,\beta) = \tilde{\phi}(\alpha,0-,\beta)$

(iv) $\frac{\partial\tilde{\phi}}{\partial y}(\alpha,0+,\beta) - \varepsilon_r \frac{\partial\tilde{\phi}}{\partial y}(\alpha,0-,\beta) = -\frac{1}{\varepsilon_0}\tilde{\rho}(\alpha,\beta)$

where $\tilde{\rho}$ is the Fourier transform of the charge distribution on the strip, and ε_0 is the free space permittivity. When applying these conditions to the Poisson equation in the transform domain, we obtain

$$G(\alpha,\beta)\,\tilde{\rho}(\alpha,\beta) = \tilde{\phi}_i(\alpha,0,\beta) + \tilde{\phi}_o(\alpha,0,\beta) \qquad (2)$$

where

$$G(\alpha,\beta) = \frac{1}{\varepsilon_0\sqrt{\alpha^2+\beta^2}\,[1 + \varepsilon_r \coth\sqrt{\alpha^2+\beta^2}\,b]} \qquad (3)$$

In equation (2), $\tilde{\phi}_i$ and $\tilde{\phi}_o$ are the transforms of the potential functions on the strip and outside of the strip at y = 0, respectively. Note that we have obtained an algebraic equation (2) rather than an integral equation which is usually derived in the space domain. This is one of the features of the method which reduce the computational effort greatly. The equation thus derived contains two unknowns, the transforms of charge on the strip $\tilde{\rho}$ and the potential outside of the strip $\tilde{\phi}_o$.

The next step is to apply Galerkin's method in the transform domain. This step eliminates one of the unknowns $\tilde{\phi}_o$ and converts the algebraic equation into a matrix equation which is subsequently solved for the unknown coefficients. The matrix equation is

$$\sum_{n=1}^{N} K_{mn} d_n = f_m \qquad m = 1,2,\ldots, N \qquad (4)$$

where

$$K_{mn} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{\zeta}_m(\alpha,\beta)\, G(\alpha,\beta)\, \tilde{\zeta}_n(\alpha,\beta)\, d\alpha d\beta$$

$$f_m = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{\zeta}_m(\alpha,\beta)\, \tilde{\phi}_i(\alpha,0,\beta)\, d\alpha d\beta$$

and $\tilde{\zeta}_m$ are the basis functions. The elimination of $\tilde{\phi}_o$ can be shown easily by the application of Parseval's relation.[4] Finally, the charge distribution is expressed in the space domain in terms of the superposition of the inverse transforms of the basis functions weighted by the coefficients d_n just obtained.

It is convenient to choose the basis functions whose inverse transforms are analytically known and have the finite support in the space domain. In the actual calculation, we have used, for the inverse of basis functions, the polynomials in x and z on the strip and zero outside, viz

$$\zeta_m(x,z) = \mathcal{F}^{-1}[\tilde{\zeta}_m(\alpha,\beta)] = \begin{cases} |x|^{k-1}\,|z|^{j-1}, & \text{on strip} \\ 0, & \text{otherwise} \end{cases}$$

$m = 1$ ($k=1$, $j=1$), $m = 2$ ($k=2$, $j=1$)...

Numerical Results

Numerical results were obtained first for the $\ell = W$ case. Figure 2 shows the results obtained for dielectric constants of 1.0 and 9.6. The capacitance is normalized by the approximate parallel-plate capacitance $\varepsilon W^2/b$. As expected, the normalized capacitance approaches one as b/W becomes small. Also, note that for the higher dielectric constant, e.g., 9.6, the capacitance approaches one more rapidly. This is also to be expected since the fringing effect decreases with increasing dielectric constant of the substrate. These results compare very well with the results of Farrar and Adams,[5] who used the point-matching method, and Reitan[6] who employed the method of sub-areas for $\varepsilon_r = 1.0$.

The calculation of the fringing or excess capacitance of a semi-infinite microstrip was as follows: As the length of the section became arbitrarily long, it is apparent that the charge distribution in the mid-section will approach that of a uniform microstrip transmission line. Hence, if we can calculate the total capacitance of this finitely long rectangular line and extract the line capacitance for a uniform line of the same width, it is possible to obtain the excess capacitance of a semi-infinite microstrip line.

The excess capacitance C_{ex} of the semi-infinite microstrip is calculated by the equation[5]

$$\lim_{\ell \to \infty} C_{ex}(\ell) = 1/2[C(\ell) - \ell C_o] \tag{5}$$

where $C(\ell)$ is the total capacitance of a rectangular section of length ℓ and width W, C_o is the line capacitance of a uniform microstrip of width W, and the factor of 1/2 accounts for the discontinuities at both ends of the strip. The length of the section is increased until $C_{ex}(\ell + \Delta\ell) \simeq C_{ex}(\ell)$. That is, the excess capacitance does not increase significantly when the length is made longer than some finite length ℓ. At this point, it can be assumed that the charge distribution has become uniform.

Figure 3 shows a plot of the capacitance $C(\ell)$ of a rectangular microstrip with W/b = 1 as a function of length. It is seen that $C(\ell)$ is a linear function of length for $\ell/W \geq 4$.

Figure 4 shows the excess capacitance calculated for a semi-infinite microstrip with $\varepsilon_r = 9.6$ over the range W/b from 0.1 to 10. The results are compared with those of Farrar and Adams; the agreement is good for W/b < 4. However, our results indicate that the lumped capacitance normalized by the width becomes a constant for wide strips. Intuitively, this can be visualized by considering the slowly varying charge distribution for a wide strip.

Conclusion

It should be noted that the method described in this paper has many advantages, one of which is its numerical efficiency. Another feature is that many other types of junctions and finite structures can be solved by the method in this paper in its present form or with slight modification.

References

1. R. Mittra and T. Itoh, "A New Method for the Analysis of the Dispersion Characteristics of Microstrip Lines," IEEE Transactions, Vol. MTT-19, pp. 47-56, 1971.

2. E. Yamashita and R. Mittra, "Variational Method for the Analysis of Microstrip Lines, IEEE Transactions, Vol. MTT-16, pp. 251-256, 1968.

3. E. Yamashita, "Variational Method for the Analysis of Microstrip-Like Transmission Lines," IEEE Transactions, Vol. MTT-16, pp. 529-535, 1968.

4. T. Itoh and R. Mittra, "Dispersion Characteristics of the Slot Lines," Electronics Letters, Vol. 7, pp. 364-365, 1971.

5. A. Farrar and A. F. Adams, "Computation of Lumped Microstrip Capacities by Matrix Methods - Rectangular Sections and End Effect," IEEE Transactions (Correspondence), Vol. MTT-19, pp. 495-97, 1971.

6. D. K. Reitan, "Accurate Determination of the Capacitance of Rectangular Parallel-Plate Capacitors," Journal of Applied Physics, Vol. 30, pp. 172-176, 1959.

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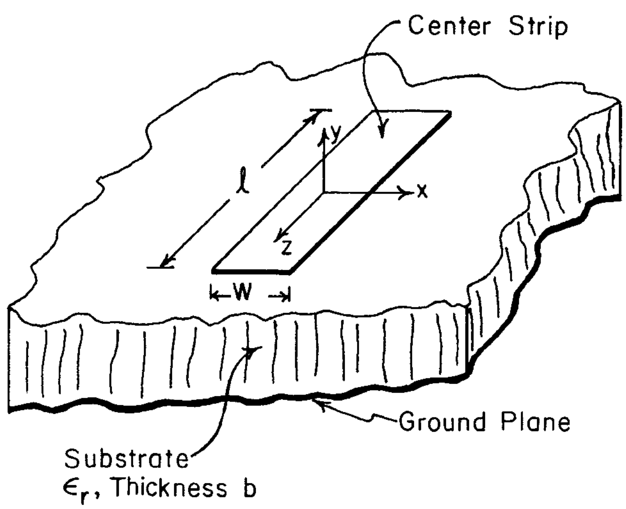

FIG. 1. FINITE SECTION OF MICROSTRIP LINE

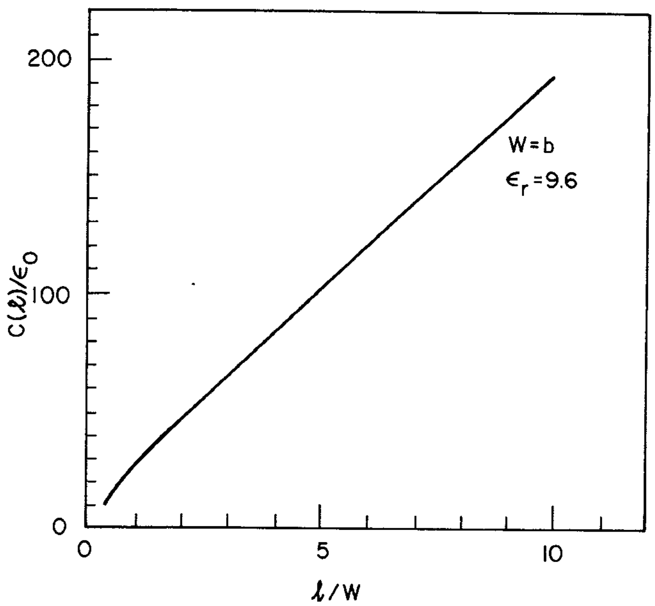

FIG. 3. CAPACITANCE OF A RECTANGULAR MICROSTRIP SECTION

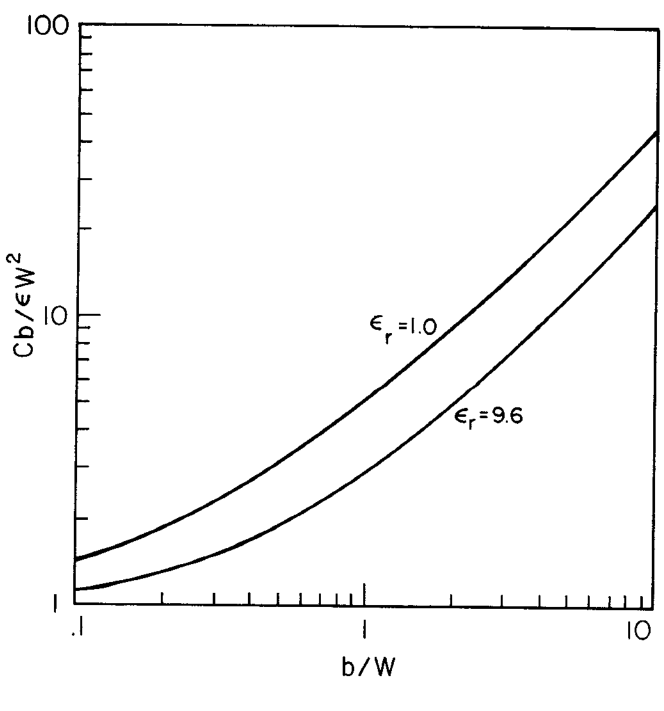

FIG. 2. CAPACITANCE OF A SQUARE SECTION OF MICROSTRIP

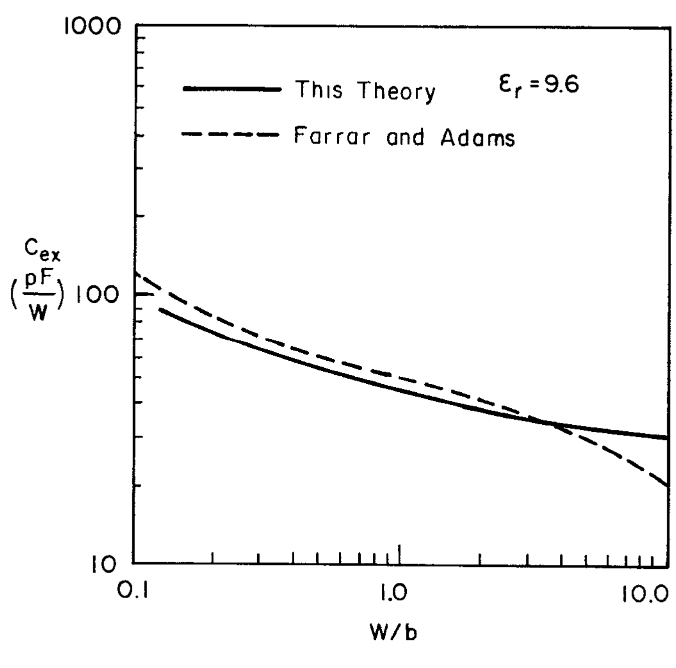

FIG. 4. EXCESS CAPACITANCE OF A SEMI-INFINITE MICROSTRIP LINE